

SOLUZIONE

a) Calcolo dell'equilibrio

$$\begin{cases} 0 = -\log \bar{x}_1 + 2 \log \bar{x}_2 - 1 \\ 0 = -\frac{\bar{x}_2}{\bar{x}_1} + 1 \end{cases} \Rightarrow \begin{cases} -\log \bar{x}_1 + 2 \log \bar{x}_2 - 1 = 0 \\ \bar{x}_1 = \bar{x}_2 \end{cases} \Rightarrow \begin{cases} \bar{x}_1 = e \\ \bar{x}_2 = e \end{cases}$$

Linearizzazione

$$\begin{cases} \delta \dot{x}_1 = -\delta u \log \bar{x}_1 - \frac{\bar{u}}{\bar{x}_1} \delta x_1 + \frac{2}{\bar{x}_2} \delta x_2 - \delta u \\ \delta \dot{x}_2 = \frac{\bar{x}_2}{\bar{x}_1^2} \delta x_1 - \frac{1}{\bar{x}_1} \delta x_2 + \delta u \end{cases} \Rightarrow \begin{cases} \delta \dot{x}_1 = -\frac{1}{e} \delta x_1 + \frac{2}{e} \delta x_2 - 2\delta u \\ \delta \dot{x}_2 = \frac{1}{e} \delta x_1 - \frac{1}{e} \delta x_2 + \delta u \end{cases}$$

$$A = \begin{bmatrix} -\frac{1}{e} & \frac{2}{e} \\ \frac{1}{e} & -\frac{1}{e} \end{bmatrix} \quad B = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \quad C = [0 \quad 1]$$

$$\text{b) } (sI - A) = \begin{bmatrix} s + \frac{1}{e} & -\frac{2}{e} \\ -\frac{1}{e} & s + \frac{1}{e} \end{bmatrix} \Rightarrow (sI - A)^{-1} = \frac{1}{e^2 s^2 + 2es - 1} \begin{bmatrix} e^2 s + e & 2e \\ e & e^2 s + e \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1} B = [0 \quad 1] \frac{1}{e^2 s^2 + 2es - 1} \begin{bmatrix} e^2 s + e & 2e \\ e & e^2 s + e \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \frac{e(es-1)}{e^2 s^2 + 2es - 1}$$

guadagno = e

$$\text{poli} = \left\{ \frac{\sqrt{2}-1}{e}, \frac{-\sqrt{2}-1}{e} \right\}, \text{ costanti di tempo} = \left\{ \frac{e}{1-\sqrt{2}}, \frac{e}{\sqrt{2}+1} \right\}$$

$$\text{zero} = \frac{1}{e}$$